**Recap:**

1. Arithmetic Progression, First term (a), Common Difference (d) and number of terms (n)

**Learning Outcomes:**

1. Geometric Progressions with for loops and append
2. Geometric Progressions with summation and counter
3. Geometric Progression (Scaling Up) and Functions
4. Geometric Progression (Scaling Down) and Functions

**Explanation Points:**

* Recap on Arithmetic Progression – pattern formed by increase of a fixed number
* Explain Geometric Progression – pattern formed by a fixed ratio between 2 successive numbers
* Application of append in a geometric progression.
* Explain on finding the sum of numbers in a geometric progression
* Explain the concept of rate (r), number of terms (n) and the first term (a) of a Geometric Progression.

**Breakdown of Lesson Plan:**

|  |  |
| --- | --- |
| Recap Lesson 4 and 5 Quiz (Arithmetic Progression) | 10 mins |
| Lesson 7.1 (Geometric Progression with for loops and Append | 20mins |
| Lesson 7.2 (Geometric Progression with Summation and Counter | 20 mins |
| Lesson 7.3 (Geometric Progression Scaling Up) with Functions | 20 mins |
| Lesson 7.4 (Geometric Progression Scaling Down) with Functions | 20 min |
| Lesson 7 Quiz | 10 mins |

*\*Note: There is a high chance of student not being able to complete on time.*

**Recap Lesson 4 and 5 Quiz (Arithmetic Progression)**

Question 1:

Jonah is studying the number of leaves on a certain species of plant. He notices the pattern over 5 days. Write a function that can help him find the expected number of leaves from the 6th day to the 30th day. Your answer should be in the format:

|  |  |
| --- | --- |
| **Days** | **Number of Leaves** |
| 1 | 6 |
| 2 | 12 |
| 3 | 18 |
| 4 | 24 |
| 5 | 30 |
| 6 | 36 |
| 7 | 42 |
| . |  |
| . |  |
| 30 | 180 |

“On day \_7\_\_, there should be \_42\_\_ leaves expected.”

.

.

.

.

.

“On day \_30\_\_, there should be \_180\_\_ leaves expected.”

Question 2:

Timothy has a garden that has a snake problem. He decides to track the number of snakes in his garden over 3 months. In the first month, he notices there were 5 snakes. In the second month, he notices there were 11 snakes, and in the third month, there were 17 snakes. He deduces that the number of snakes in his garden follows an Arithmetic Sequence. Help him determine the values of a and d (in the Arithmetic Sequence).

Using the information above, write a function to help him find the number of snakes in 6 months. Print your answer in the format:

“After 6 months, Timothy will have a total of \_\_120\_\_ snakes in his garden.”

|  |  |
| --- | --- |
| **Month** | **Number of Leaves** |
| 1 | 5 |
| 2 | 11 |
| 3 | 17 |
| 4 | 23 |
| 5 | 29 |
| 6 | 35 |

**Lesson 7.1**

**As we further study different number sequences, we notice that Arithmetic Progression does not fit all the known number sequences.**

|  |  |
| --- | --- |
| **Term** | **Value** |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** | **4** |
| **5** |  |
| **6** |  |

**Recap in Lesson 4, we looked at Arithmetic Progression. The number increases by a fixed number each time. In the example below, the fixed number is 2.**

**What happens if we scale the numbers instead? This is where we bring in the concept of a Geometric Sequence.**

**A Geometric Progression is a number sequence such that the ratio of any two successive numbers is a constant – in this case, the constant is 2. Let us look at such an example:**

**2, 4, 8, 16, 32, 64…**

|  |  |
| --- | --- |
| **Term** | **Value** |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |
| **6** |  |

**This Progression looks rather difficult to understand. Let us see if we can find a common ratio and pattern that would suit it.**

**Lesson 7.1**

**Example a**

**From the above, we can see the common ratio is 2. The answer from the previous line is used to multiply by 2. Translating that below:**

Output

|  |  |
| --- | --- |
| 1 | lastterm=1 |
| 2 | for i in range(0,6): |
| 3 | nextterm=lastterm\*2 |
| 4 | print(lastterm) |
| 5 | lastterm=nextterm |

|  |  |
| --- | --- |
| *1* |  |
| *2* |  |
| *3* |  |
| *4* |  |
| *5* |  |
| *6* |  |

**Example b**

Output

|  |  |
| --- | --- |
| 1 | lastterm=1 |
| 2 | for i in range(0,6): |
| 3 | nextterm=lastterm\*2 |
| 4 | print(lastterm) |
| 5 | lastterm=nextterm |

|  |  |
| --- | --- |
| *1* |  |
| *2* |  |
| *3* |  |
| *4* |  |
| *5* |  |
| *6* |  |

**Question: What happens if we swap lines 4 and 5 in example a and b? Explain the code sequence**

**Example c**

**What if we change the lastterm to 2?**

Output

|  |  |
| --- | --- |
| 1 | lastterm=2 |
| 2 | for i in range(0,6): |
| 3 | nextterm=lastterm\*2 |
| 4 | print(lastterm) |
| 5 | lastterm=nextterm |

|  |  |
| --- | --- |
| *1* |  |
| *2* |  |
| *3* |  |
| *4* |  |
| *5* |  |
| *6* |  |

**Question: How do we get the same output as example a and b with lastterm set at 2?**

**Lesson 7.1**

**Recap on append:**

Output

|  |  |
| --- | --- |
| 1 | sequence=[2,4,6,8] |
| 2 | sequence.append(10) |
| 3 | print(sequence) |

|  |  |
| --- | --- |
| *1* |  |

Output

|  |  |
| --- | --- |
| 1 | sequence=[2,4,6,8] |
| 2 | for i in range(10,20,2) |
| 3 | sequence.append(i) |
| 4 | print(sequence) |

|  |  |
| --- | --- |
| *1* |  |

Task 1

Similar to example a and b, now, I would like the output of the GP sequence in the format below. You will need to use indexing and append. (Complete line 2)

Output

|  |  |
| --- | --- |
| 1 | sequence=[8] |
| 2 | for i in range ( ): |
| 3 | nextterm=sequence[i] \* 2 |
| 4 | sequence.append (nextterm) |
| 5 |  |
| 6 | print(sequence) |

|  |  |
| --- | --- |
| *1* | [8,16,32,64,128,256,512] |

Task 2

Continuing from the example a and b, now, I would like the output of the GP sequence in the format below. You will need to use append

Output

|  |  |
| --- | --- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |

|  |  |
| --- | --- |
| *1* | [2,4,8,16,32,64,128,256,512,1024] |

**Lesson 7.1**

Task 3

In this task, complete the code to get the output below. It is a geometric progression of 4. The lastterm is set at 1.

Output

|  |  |
| --- | --- |
| 1 | lastterm= 1 |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 4 |
| *2* | 16 |
| *3* | 64 |
| *4* | 256 |
| *5* | 1024 |
| *6* | 4096 |

Task 4

In this task, complete the code to get the output below.It is a geometric progression of 4. The lastterm is set at 4.

Output

|  |  |
| --- | --- |
| 1 | lastterm= 4 |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 4 |
| *2* | 16 |
| *3* | 64 |
| *4* | 256 |
| *5* | 1024 |
| *6* | 4096 |

Task 5

In this task, using for loops and append, write the code to get the output below.

Output

|  |  |
| --- | --- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

|  |  |
| --- | --- |
| *1* | [5,25,125,625,3125] |

**Lesson 7.1**

Task 6

Similar to Task 3 but take note on the indentation.

In this task, complete the code to get the 3rd output of a geometric progression starting with 4. The lastterm is set at 1.

Output

|  |  |
| --- | --- |
| 1 | lastterm= 1 |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 64 |

Task 7

Similar to Task 3 but take note on the indentation.

In this task, complete the code to get the 3rd output of a geometric progression of 4 starting with 4. The lastterm is set at 4.

Output

|  |  |
| --- | --- |
| 1 | lastterm= 4 |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 64 |

**Task 6 and 7, Explain the code sequence and write the key takeaways.**

**Lesson 7.1**

Task 8

In this task, complete the code to get the 6th output of a geometric progression of 3. It is a geometric progression starting with 3. [3,9,27,81,243,729]

Output

|  |  |
| --- | --- |
| 1 | lastterm= |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 729 |

Task 9

In this task, complete the code to get the 10th output of the geometric progression below.

[1,6,36,216,1296,……]

Output

|  |  |
| --- | --- |
| 1 | lastterm= |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 10077696 |

Task 10

In this task, complete the code to get the 12th output of the geometric progression below.

[1,8,64,512,4096,……]

Output

|  |  |
| --- | --- |
| 1 | lastterm= |
| 2 | for i in range( ): |
| 3 |  |
| 4 |  |
| 5 |  |

|  |  |
| --- | --- |
| *1* | 8589934592 |

**Lesson 7.2**

In this section, we would like to find the sum of numbers in a geometric sequence.

Example

Print the sum of the numbers in the sequence [2,4,8,16,32,64] as well as the sequence.

Output

|  |  |
| --- | --- |
| 1 | lastterm=1 |
| 2 | sum1=0 |
| 3 | for i in range(0,6): |
| 4 | nextterm=lastterm\*2 |
| 5 | lastterm=nextterm |
| 6 | sum1=sum1+lastterm |
| 7 | print(lastterm) |
| 8 | print(sum1) |

|  |  |
| --- | --- |
| *1* |  |
| *2* |  |
| *3* |  |
| *4* |  |
| *5* |  |
| *6* |  |
| *7* |  |

Task 1

Print the sum of the numbers in the sequence [ 3,9,27,81,243,729,2187,6561]

Output

|  |  |
| --- | --- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |

|  |  |
| --- | --- |
| *1* | 9840 |

**Lesson 7.3**

In this section, we will use functions with our Geometric Progression in increasing number

Task 1:

Rancher Randy rears rabbits at his farm. Every month, the number of rabbits on his farm doubles. This follows a Geometric Progression, where the first term is 2, and the common ratio is 2. Write a function that reflects this Geometric Progression, and output the first 10 terms of this Geometric Progression in the following format: (Complete line 4)

“On Month \_\_0 \_\_ Rancher Randy has \_\_2\_\_ rabbits.”

“On Month \_\_1 \_\_ Rancher Randy has \_\_4\_\_ rabbits.”

“On Month \_\_2 \_\_ Rancher Randy has \_\_8\_\_ rabbits.”

“On Month \_\_3 \_\_ Rancher Randy has \_\_16\_\_ rabbits.”

.

.

“On Month \_\_9 \_\_ Rancher Randy has \_\_1024\_\_ rabbits.”

|  |  |
| --- | --- |
| 1 | def G\_P(a,r,n): |
| 2 | first = a |
| 3 | for num in range(n): |
| 4 | print( ) |
| 5 | nextterm=first x r |
| 6 | first=nextterm |
| 7 |  |
| 8 | G\_P(2,2,10) |

**Task 1, Explain the code sequence and write the key takeaways.**

a = first term

r = rate of increase

n= number of outputs

**Lesson 7.3**

Task 2:

Jamie is studying amoebas and cell division again. Her culture starts with 5 amoeba multiplies by 2 every cycle. She recognizes that the amoeba cell division process follows a Geometric Progression. Write a function that reflects this Geometric Progression and output the first 12 terms of this Geometric Progression in the format below.

“After \_\_0\_\_ cycles, there will be \_\_5\_\_ amoebas.”

“After \_\_1\_\_ cycles, there will be \_\_10\_\_ amoebas.”

“After \_\_2\_\_ cycles, there will be \_\_20\_\_ amoebas.”

“After \_\_3\_\_ cycles, there will be \_\_40\_\_ amoebas.”

.

.

.

“After \_\_11\_\_ cycles, there will be \_\_10240\_\_ amoebas.”

Fill in the following and complete the code below

a = first term =

r = rate of increase =

n= number of outputs =

|  |  |
| --- | --- |
| 1 | def G\_P(a,r,n): |
| 2 | first = a |
| 3 | for num in range(\_\_\_\_): |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 | G\_P( |

**Lesson 7.3**

Task 3:

James borrows some money from a bank. The interest rate of the bank is 5% per annum (multiply by 1.05). He plans to borrow $10 for his business. This follows a Geometric Progression. Tell your teacher what the first term and the common ratio is. Write a function that reflects this Geometric Progression.

James wants to know how much he would owe after 7 years. Print your answer in the format:

“After \_\_7\_\_ years, James will owe $ \_\_14.071004\_\_.”

Task 4:

Kane is studying the impact of a viral meme. He wants to know how many people it can reach in 10 hours. He knows that his meme would be shared once every 1 hr. Every share will reach 5 other people. This follows a Geometric Progression starting from 1 audience. Write a function to represent this Geometric Progression. Print your answer in this format:

“After \_0\_ hours, Kane’s meme will reach \_\_1\_\_ audiences.”

“After \_1\_ hours, Kane’s meme will reach \_\_5\_\_ audiences.”

“After \_2\_ hours, Kane’s meme will reach \_\_25\_\_ audiences.”

“After \_3\_ hours, Kane’s meme will reach \_\_125\_\_ audiences.”

.

.

“After \_10\_ hours, Kane’s meme will reach \_\_9765625\_\_ audiences.”

After finding the new audiences reached, Kane wants to know how many people actually saw his meme. Edit your function to help him find the number of people who saw his meme. Print your answer at the end in the following format:

“Kane’s meme will reach \_\_61035155\_\_ people in 10 hours.”

**Lesson 7.4**

In this section, we will use functions with our Geometric Progression in decreasing number

Task 1:

Rancher Badluck rears organisms at his farm. He has 100 organisms at the start. Every month, the number of organims on his farm dies at a speed double the number from the last month. This follows a Geometric Progression, where the first term is 100, and the common ratio is 2. Write a function that reflects this Geometric Progression, and output the first 10 terms of this Geometric Progression in the following format: (Complete line 4). Note: This is a special type of organism that can exist not in whole

“On Month \_\_0 \_\_ Rancher Randy has \_\_100\_\_ organisms.”

“On Month \_\_1 \_\_ Rancher Randy has \_\_50\_\_ organisms.”

“On Month \_\_2 \_\_ Rancher Randy has \_\_25\_\_ organisms.”

“On Month \_\_3 \_\_ Rancher Randy has \_\_12.5\_\_ organisms.”

.

.

“On Month \_\_9 \_\_ Rancher Randy has \_\_0.1953125\_\_ organisms.”

|  |  |
| --- | --- |
| 1 | def G\_P(a,r,n): |
| 2 | first = a |
| 3 | for num in range(n): |
| 4 | print( ) |
| 5 | nextterm=first / r |
| 6 | first=nextterm |
| 7 |  |
| 8 | G\_P(100,2,10) |

**Task 1, Explain the code sequence and write the key takeaways.**

a = first term

r = rate of increase

n= number of outputs

**Lesson 7.4**

Task 2

Scientist Scarlet is studying the rate of decay of a copper sample. She knows that the sample has a half-life of 1 hour (i.e. every 1 hour, the weight of the sample will be halved). She thinks that this process follows a simple Geometric Progression. Her copper sample has a weight of 3000 grams. Write a function that shows the weight of the copper sample after 5 hours. Print your answer in the format:

“After \_\_0\_\_ hours, the Copper Sample weighs \_3000\_\_ grams.”

“After \_\_1\_\_ hours, the Copper Sample weighs \_1500\_\_ grams.”

“After \_\_2\_\_ hours, the Copper Sample weighs \_750\_\_ grams.”

.

.

.

“After \_\_5\_\_ hours, the Copper Sample weighs \_93.75\_\_ grams.”

Fill in the following and complete the code below

a = first term =

r = rate of increase =

n= number of outputs =

|  |  |
| --- | --- |
| 1 | def G\_P(a,r,n): |
| 2 | first = a |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 | G\_P( |

**Lesson 7.4**

Task 3:

The traditional A-series paper (A0, A1, etc.) actually follows a Geometric Progression with a rate of decrease of 1.4144. The length of an A0 paper is 1190 mm approximately. Write a function to represent this Geometric Progression. Print the length of the different paper sizes in the format:

“A\_0\_ size paper has a length of \_\_\_ mm approximately.”

“A\_1\_\_ size paper has a length of \_\_\_ mm approximately.”

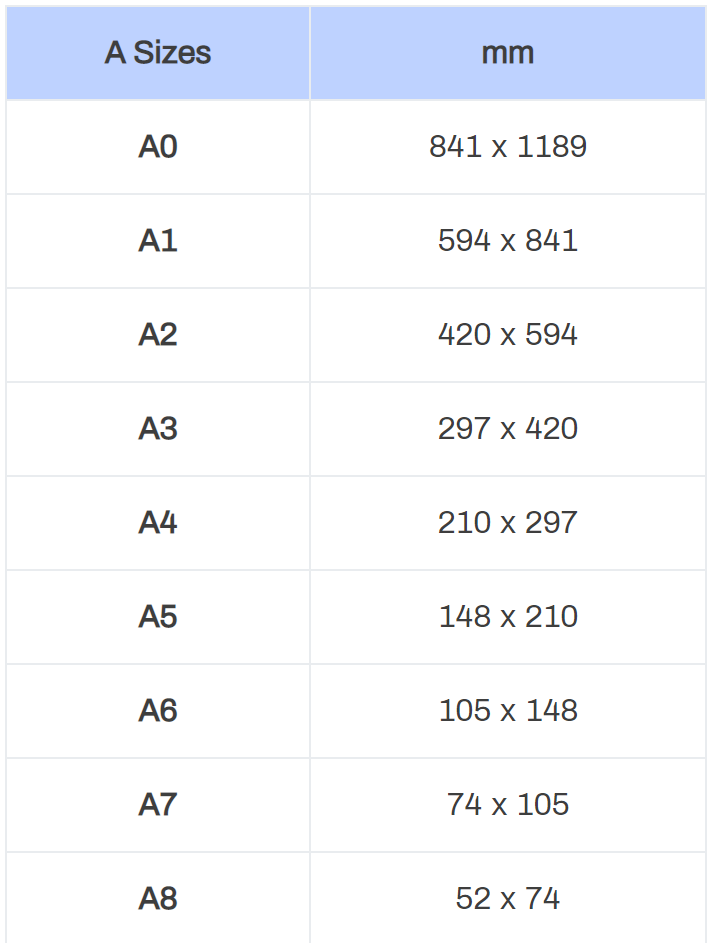
“A\_2\_ size paper has a length of \_\_\_ mm approximately.”

.

.

.

“A\_8\_\_ size paper has a length of \_\_\_ mm approximately.”



**Lesson 7.4**

Task 4:

John has a bag of 4096 sweets. He decides to give a quarter of his remaining sweets to every child he meets along the way. This follows a Geometric Progression. Write a function that represents this Geometric Progression, and find the number of sweets remaining after John meets 6 kids. Print your answer in the format:

“After meeting kid number \_\_\_, John will have \_\_\_ sweets remaining.”

**End of Lesson 7 Quiz:**

Question 1:

James is interested to save his money in a bank. He has a total of $10000 that he wishes to save. Help James pick the best bank to save with.

Bank A follows an Arithmetic Progression, that pays $600 per year for the $10000 that James wishes to save. Write a function that represents this Arithmetic Progression. Print the amount of money James will have in his bank after 10 years. Your answer should be in the format:

“After \_\_\_ years, James will have $\_\_\_ in the bank if he chooses Bank A.”

Bank B follows a Geometric Progression and increases the amount of money in the bank by 5% (multiply by 1.05). Write a function that represents this Geometric Progression. Print the amount of money James will have in his bank after 10 years. Your answer should be in the format:

“After \_\_\_ years, James will have $\_\_\_ in the bank if he chooses Bank B.”

Between the two banks, which do you think is better? Discuss with your teacher.

**Optional:** For students who are faster, they can learn to use exponent with geometric progression.

**A Geometric Progression is a number sequence such that the ratio of any two successive numbers is a constant. Let us look at such an example:**

**4, 8, 16, 32, 64…**

|  |  |
| --- | --- |
| **Term** | **Value** |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |

**This Progression looks rather difficult to understand. Let us see if we can find a common ratio that would suit it.**

**From the table above, we can find a nth term by using the formula . This means that for each term, we multiply the previous term by a constant 2. Hence, we can reflect this Geometric Progression by using a simple for loop.**

|  |  |
| --- | --- |
| ***1*** | **def G\_P(a,r,n):** |
| ***2*** | **first = a** |
| ***3*** | **for num in range(n):** |
| ***4*** | **print(first)** |
| ***5*** | **first\*=r** |
| ***6*** |  |
| ***7*** | **G\_P(4,2,5)** |

**Question: What happens if we swap lines 4 and 5?**

**Optional:** For students who are faster, they can learn to use exponent with geometric progression.

**Similar to Arithmetic Progressions, Geometric Progressions can also move in the opposite direction. Rather than scaling the numbers up, we can instead scale the numbers down. Let us take a look at some examples.**

**A Geometric Progression is a number sequence such that the ratio of any two successive numbers is a constant. Let us look at such an example:**

**8, 4, 2, 1, 0.5, 0.25…**

|  |  |
| --- | --- |
| **Term** | **Value** |
| **1** |  |
| **2** |  |
| **3** |  |
| **4** |  |
| **5** |  |

**From the table above, we can find a nth term by using the formula 3 + 2 x (n-1). Hence, we can write a function to find the 5th and 1st terms.**

**From the table above, we can find a nth term by using the formula . This means that for each term, we divide the previous term by a constant 2. Hence, we can reflect this Geometric Progression by using a simple for loop.**

|  |  |
| --- | --- |
| ***1*** | **def G\_P(a,r,n):** |
| ***2*** | **first = a** |
| ***3*** | **for num in range(n):** |
| ***4*** | **print(first)** |
| ***5*** | **first\*=r** |
| ***6*** |  |
| ***7*** | **G\_P(8,0.5,5)** |

**Question: What happens if we swap lines 4 and 5?**